

Quantum tunneling radiation from self-dual black holes

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Abstract

We calculate the Hawking temperature for a self-dual black hole in the context of quantum tunneling formalism.

1. Introduction

Black holes are putative objects whose gravitational fields are so strong that no physical bodies or signals can break free of their pull and scape. In the seventies, through the Hawking demonstration that all black holes emit blackbody radiation [1], the study of black holes obtained a position of significance going far beyond astrophysics, since, in the presence of a black hole strong gravitational field, the quantum nature of spacetime must be manifested.

One approach to quantum gravity, Loop Quantum Gravity (LQG), has given rise to models that afford a description of the quantum spacetime revealed by a black hole. Actually, string theory and loop quantum gravity, lately, showed that the origin of the black-hole thermodynamics must be related with the quantum structure of the spacetime, bringing together the developments in black-hole physics and the improvement of our understanding on the nature of the spacetime in quantum gravity regime

In particular, in loop quantum gravity context, a black hole metric, known as the loop black hole (LBH), or self-dual black hole [23, 36], has the interesting property of self-duality that removes the black hole singularity and replaces it with another asymptotically flat region. The issue of the thermodynamical of this kind of black hole has been investigated in [32, 33, 34], and the dynamical aspects of the collapse and evaporation were studied in [35] where the habitual Hawking formalism to derive the black hole thermodynamical properties was used.

By the way, since Hawking proved that black holes can radiate thermally [1], in a way that these objects are kinds of thermal system and have thermodynamic relations among the quantities describing them, several efforts in order to derive the temperature and entropy of black holes have been done via various methods.

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While Hawking used the quantum field theory in curved spacetime in his original paper [1], there exist other methods which give the same predictions [2, 3, 4, 5]. Although all these methods have been successful in deriving the temperature or the entropy of certain types of black holes, it is not satisfactory in the sense that they do not reveal the dynamical nature of the radiation process, since the background geometry is fixed mostly in these scenarios, including the Hawking's scenario.

In recent years, a semiclassical method has been developed viewing Hawking radiation as a tunneling phenomena across the horizon [6, 7, 8]. The essential idea is that the positive energy particle created just inside the horizon can tunnel through the geometric barrier quantum mechanically, and it is observed as the Hawking flux at infinity. The black hole tunneling method has a lot of strengths when compared to other methods for calculating the temperature. To cite some of these strengths, we have that the tunneling method is a particularly interesting method for calculating black hole temperature since it provides a dynamical model of the black hole radiation. Besides, the calculations in this approach are straightforward and relatively simple, and the tunneling method is robust in the sense that it can be applied to a wide variety of exotic spacetimes [9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 18, 19, 20, 21].

In this paper, we will use the tunneling formalism to investigate the thermodynamic properties of self-dual black holes.

2. Self-dual black holes

Loop quantum gravity is based on the formulation of classical general relativity, which goes under the name of new variables, or Ashtekar variables, that is in terms of an $su(2)$ 3-dimensional connection A and a triad E . The basis states of LQG then are closed graphs the edges of which are labeled by irreducible $su(2)$ representations and the vertices by $su(2)$ intertwiners. One of the most significant result of loop quantum gravity is the discovery that certain geometrical quantities, in particular area and volume, are represented by operators that have discrete eigenvalues.

The regular black hole metric that we will be using is derived from a simplified model of LQG [23]. In this context, the quantum gravitational corrected metric provided by LQG is

$$ds^2 = -G(r)dt^2 + F(r)^{-1}dr^2 + H(r)d\Omega^2 \quad (1)$$

with

$$d\Omega^2 = d\theta^2 + \sin^2 \theta \phi^2 \quad (2)$$

The metric functions are given by

$$G(r) = \frac{(r - r_+)(r - r_-)(r - r_*)}{r^4 + a_0^2}, \quad (3)$$

$$F(r) = \frac{(r - r_+)(r - r_-)r^4}{(r + r_*)^2(r^4 + a_0^2)}, \quad (4)$$

and

$$H(r) = r^2 + \frac{a_0^2}{r^2} \quad (5)$$

where

$$r_+ = \frac{2Gm}{c^2} \quad ; \quad r_- = \frac{2Gm}{c^2} P^2$$

$$r_* = \sqrt{r_+ r_-} = 2mP \quad (6)$$

$$P = \frac{\sqrt{1 + \epsilon^2} - 1}{\sqrt{1 + \epsilon^2} + 1} \quad ; \quad a_0 = \frac{A_{\min}}{8\pi} = \frac{\sqrt{3}}{2} \gamma \beta R_P^2 \quad (7)$$

In the next section, we will use the tunneling formalism to derive the Hawking temperature for a black hole described by the metric (1)

3. Tunneling from self-dual black holes

The first black hole tunneling method developed was the Null Geodesic Method used by Parikh and Wilczek [6, 8], which followed from the work by Kraus and Wilczek [24, 25, 26]. The other approach to black hole tunneling is the Hamilton-Jacobi ansatz used by Angheben et al, which is an extension of the complex path analysis of Padmanabhan et al [22, 27, 28, 29]. In this paper, we will work with this method, since it is more direct. Our calculations, using the Hamilton-Jacobi method, involves consideration of an emitted scalar particle, ignoring its self-gravitation, and assumes that its action satisfies the relativistic Hamilton-Jacobi equation.

To begin with, we have that, near the black hole horizon, the theory is dimensionally reduced to a 2-dimensional theory [30, 31] whose metric is just the $(t - r)$ sector of the original metric while the angular part is red-shifted away. Consequently the near-horizon metric has the form

$$ds^2 = -G(r)dt^2 + F(r)^{-1}dr^2. \quad (8)$$

Moreover, the effective potential vanishes and there are no grey-body factors. However, the self-consistency of the approach can be seen by recalling that the emission spectrum obtained from these modes is purely thermal. This justifies ignoring the grey-body factors.

Now, consider the Klein-Gordon equations

$$\hbar^2 g^{\mu\nu} \nabla_\mu \nabla_\nu \phi - m^2 \phi = 0 \quad (9)$$

under the metric given by (8)

$$-\partial_t^2 \phi + \Lambda \partial_r^2 \phi + \frac{1}{2} \Lambda' \partial_r \phi - \frac{m^2}{\hbar^2} G \phi = 0 \quad (10)$$

where $\Lambda = F(r)G(r)$

Since the typical radiation wavelength is of the order of the black hole size, one might doubt whether a point particle description is appropriate. However, when the outgoing wave is traced back towards the horizon, its wavelength, as measured by local fiducial observers, is ever-increasingly blue-shifted. Near the horizon, the radial wavenumber approaches infinity and the point particle, or WKB, approximation is justified [6].

In this way, taking the standard WKB ansatz

$$\phi(r, t) = e^{-\frac{i}{\hbar} S(r, t)}, \quad (11)$$

one can obtain the relativistic Hamilton-Jacobi equation with the limit $\hbar \rightarrow 0$,

$$(\partial_t S)^2 - \Lambda (\partial_r S)^2 - m^2 = 0 \quad (12)$$

We seek a solution of the form

$$S(r, t) = -\omega t = W(r) \quad (13)$$

Solving for $W(r)$ yields

$$W = \int \frac{dr}{\Gamma} \sqrt{\omega^2 - m^2 G} \quad (14)$$

where $\Gamma = \Lambda^{1/2}$

In this point, we will adopt the proper spatial distance,

$$d\sigma = \frac{dr^2}{\Gamma(r)} \quad (15)$$

and, by taking the near horizon approximation

$$\Gamma(r) = \Gamma'(r_H)(r - r_H) + \dots \quad (16)$$

we find that

$$\sigma = 2 \frac{\sqrt{r - r_H}}{\Gamma'(r_H)} \quad (17)$$

where $0 < \sigma < \infty$.

In this way, the spatial part of the action function reads

$$\begin{aligned} W &= \frac{2}{\Gamma'(r_H)} \int \frac{d\sigma}{\sigma} \sqrt{\omega^2 - \frac{\sigma^2}{4} m^2 G'(r_H) \Gamma'(r_H)} \\ &= \frac{2\pi i \omega}{\Gamma'(r_H)} + \text{real contribution} \end{aligned} \quad (18)$$

In this way, the Hawking temperature for the self-dual black hole is given by

$$T_H = \frac{\omega}{ImS} = \frac{(2m)^3(1-P^2)}{4\pi[(2m)^4 + a_0^2]} \quad (19)$$

The expression above for Black hole temperature has been found by [32, 33, 34]. Moreover, in the limit of m large it corresponds to the Hawking temperature, but goes to zero for $m \rightarrow 0$.

Moreover, the tunneling probability for a particle with energy ω is given by

$$\Gamma \simeq Exp[-2ImS] = Exp\left\{-\frac{\pi\omega[(2m)^4 + a_0^2]}{m^3(1-P^2)}\right\} \quad (20)$$

and the black hole entropy in this framework is given by [32, 33, 34]

$$S = \frac{4\pi(1+P)^2}{(1-P^2)} \left[\frac{16m^4 - a_0^2}{16m^2} \right] \quad (21)$$

4. Conclusions

In this work, we have used the Hamilton-Jacobi version of the tunneling formalism to derive the temperature and entropy of a self-dual black hole. The result found out corresponds to that previously obtained by [32, 33, 34], where the usual Hawking calculation was applied. The expression found out to the black hole temperature depends on the quantum of area a_0 .

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